New approaches in Eurocode 3 – efficient global structural design
Part 1: 3D model based analysis using general beam-column FEM

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Abstract

The new versions of the EN 1993-1-1 (EC3-1-1) and the EN 1993-1-5 (EC3-1-5) standards have introduced the general method designing beam-column structures; see [1] and [2]. The design method requires 3D geometric model and finite element analysis. In a series of papers we present this general design approach. The parts of the series are the following:

- Part 0: An explanatory introduction
- Part 1: 3D model based analysis using general beam-column FEM
- Part 2: Resistances of cross-sections using generalized cross-sectional models
- Part 3: Resistances of structural members using general method
- Part 4: Special issues of the 3D model based design method

Present paper deals with the general beam-column finite element analysis which is the fundamental tool of the general design approach are specified in the Eurocode 3.

1. The general beam-column finite element

1.1 Degrees of freedom and internal forces

In design practice more types of finite elements are used. The *beam-column* type element is axially compressed and bended around the strong or/and about the weak axes of the cross-section. The element is *general* if the following conditions are met:

- the shape of the cross-section is arbitrary (open or closed)
- the walls of the cross-section are relatively thin (thin-walled cross-section)
- the equilibrium equations are geometrically nonlinear and contain the warping effect (Wagner effect)

The above conditions are satisfied by Rajasekaran's element [3] which has 14 degrees of freedom. **Fig.1** shows the local system and the stress resultants of the element. The **u** axis coincides to the centroid, while the **v** and **w** axes are the strong and the weak axes of the cross-section, respectively. The stress resultants at the **j** and **k** ends of the element are denoted as:

$\mathbf{N}$	axial force
$\mathbf{T}_v, \mathbf{T}_w$	shear forces
$\mathbf{M}_v, \mathbf{M}_w$	bending moments
$\mathbf{M}_u$	torsional moment
$\mathbf{B}$	bimoment

It can be seen that the normal force and the bending moments are considered in the centroid while the shear forces, the torsional moment and the bimoment in the shear centre of the cross-section.

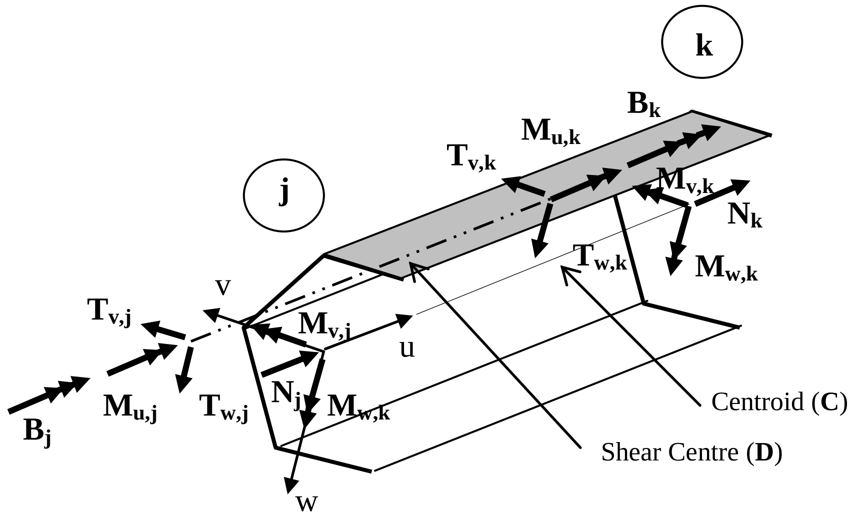


Fig. 1. Location of the stress resultants of the general beam-column finite element

However, the displacements and the stress resultants can be written in vector form:

$$\underline{u} = \lfloor u_j\ v_j\ w_j\ \theta_{u,j}\ \theta_{v,j}\ \theta_{w,j}\ \theta_{uu,j}\quad u_k\ v_k\ w_k\ \theta_{u,k}\ \theta_{v,k}\ \theta_{w,k}\ \theta_{uu,k} \rfloor \tag{1}$$

$$\underline{f} = \lfloor N_j\ T_{v,j}\ T_{w,j}\ M_{u,j}\ M_{v,j}\ M_{w,j}\ B_j\quad N_k\ T_{v,k}\ T_{w,k}\ M_{u,k}\ M_{v,k}\ M_{w,k}\ B_k \rfloor \tag{2}$$

1.2 The matrix equilibrium equation

Rajasekaran [1] derived the matrix equilibrium equation of the general element in explicit form,

$$\left(\underline{\underline{K}}_s + \underline{\underline{K}}_g\right) \times \underline{u} = \underline{f} \tag{3}$$

where $\underline{\underline{K}}_s$ is the flexural and $\underline{\underline{K}}_g$ is the geometric stiffness matrix. The stiffness matrices were derived from the virtual work equation of the element:

$$\int_l (\sigma\delta\varepsilon + \tau_{vu}\delta\gamma_{vu} + \tau_{wu}\delta\gamma_{wu})t\,ds = \int_s (\sum_d f_d \delta u_d)ds \tag{4}$$

In Eq. (4) the left hand side expresses the work of the internal stress on the appropriate virtual strain, while the right hand side expresses the work of the surface forces on the appropriate virtual displacements. At left hand side l denotes the length of the element, t is the appropriate wall thickness and s is the tangent coordinate. Furthermore, $\sigma = \sigma_u$ is the normal stress and $\delta\varepsilon$ is the corresponding virtual normal strain, τ_{vu} and τ_{wu} are the components of shearing stress and $\delta\gamma_{vu}$ and $\delta\gamma_{wu}$ are the corresponding virtual shearing strains at an arbitrary point on the counter of the element. The index d at right hand side denotes the degrees of freedom of the nodes, as it is given in Eq. (1). However, the $\underline{\underline{K}}_s$ flexural stiffness matrix is expressed in terms of the geometrical properties of the element, while $\underline{\underline{K}}_g$ is expressed in terms of the actual stress resultants such as

$$\begin{array}{ll} N & \text{axial force} \\ T_v, T_w & \text{shear forces} \\ M_v, M_w & \text{bending moments} \end{array} \tag{5}$$

Furthermore, the geometric stiffness matrix depends on the Wagner coefficient which can be written in the following general form:

$$\overline{K} = \int_s a^2 \sigma t ds \tag{6}$$

where a is the distance of the counter point of the cross-section to the shear centre. The details can be found for example in [3].

1.3 The special capabilities of the general element

The geometric stiffness matrix of the traditional 12 DOF element takes the effect of the axial force on the bending moments, but neglects the following effects:

- interaction between the bending and torsional moments
- effect of the axial stress resultant on the torsion (Wagner effect)

The 14 DOF general beam-column element is geometrically nonlinear (second order) and can take the above effects into consideration. However the general element is appropriate to compute the torsional behavior following Vlasov's theory. Practically, by this element we can compute the warping effect as well as the flexural, the torsional and the lateral torsional buckling modes, furthermore any interactions of these buckling modes.

2. Analysis of simply supported structural members

2.1 Compatibility condition for warping

The compatibility of warping may be ensured by the following condition at any node of the finite element model of any structural member (see **Fig. 2**):

$$\sum_{i=1,2} B_i = 0 \tag{7}$$

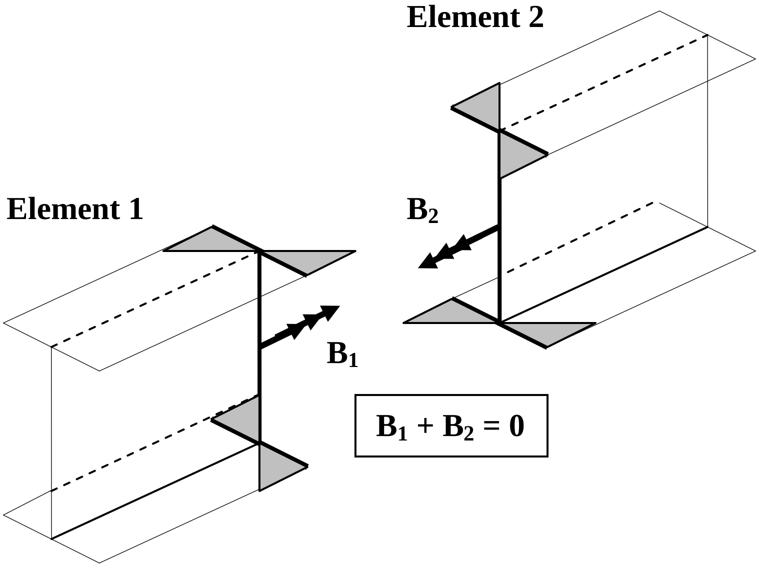


Fig. 2. Compatibility condition for warping at the joints of the general finite element model

Eq. (7) ensures that the sum of the bimoments (B_i) in the joints will be zero. If the cross-section is uniform and the member is straight, the method gives exact solution.

2.2 Modeling

The second order stress analysis and the global stability analysis which includes lateral torsional buckling of uniform structural members can be evaluated by a simple model which contains 4 to 8 general beam-column finite elements (**Fig. 3).** At any node of the model there are 7 degrees of freedom. The 7^{th} degree theoretically means the speed of the torsional deflection of the reference axis. However, any degree of the model may be restrained. Basically the 7^{th} degrees (warping) of the model supports can be restrained ($\boldsymbol{\theta_{uu}=0}$) or can be free as normally. The support model of the simply supported 3D member is defined in **Table 1**.

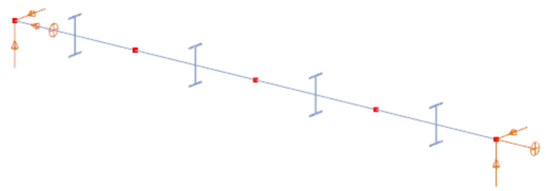


Fig. 3. The FE model of the simple supported member

member end	degrees of freedom						
	u	**v**	**w**	$\boldsymbol{\theta_u}$	$\boldsymbol{\theta_v}$	$\boldsymbol{\theta_w}$	$\boldsymbol{\theta_{uu}}$
left	**fix**	**fix**	**fix**	**fix**	free	free	free
right	free	**fix**	**fix**	**fix**	free	free	free

Tab. 1. Support model of the simple supported structural member (see Fig. 3)

2.3 Examples

The properties of a simply supported structural member are the following:
- length: 6.000 mm
- cross-section: welded I section (flanges: 200-12; web plate: 412-8)
- elastic moduli: 210.000 N/mm^2

First, let us compute the displacements and the stress resultants of the member which is loaded by concentrated torsional moment and uniform compressive force (**Example 1**). Secondly, let us compute the critical load amplifier os the member which is loaded by concentrated transverse force and uniform compressive force (**Example 2**). Let us analyze the models with the ConSteel 4.0 software [4] which uses the Rajasekaran's general beam-column element which was described in Section 1 of this paper. We will denote this element as *Beam7*. Normally, any structural member into will be distributed into eight finite elements. We will verify the analysis with independent shell finite element method where we will use a geometrically nonlinear triangular shell element with 3 nodes. We will denote this element as *Shell3*. However, we will use end stiffeners in the models to avoid local buckling as well as distorsion. We will use 2 mm thick end plates which have negligible effect on the analysis.

2.3.1 **Example 1**: Stress analysis

The load model has two components: 5 kN concentrated torsional moment at the middle cross-section and 300 kN compressive axial force at the right end of the member (Fig. 4). The specific results of the analysis which was carried out on the *Beam7* model are shown in Tab. 2. We analyzed the structural member using *Shell3* FE model. The appropriate results are shown in the Tab. 2.

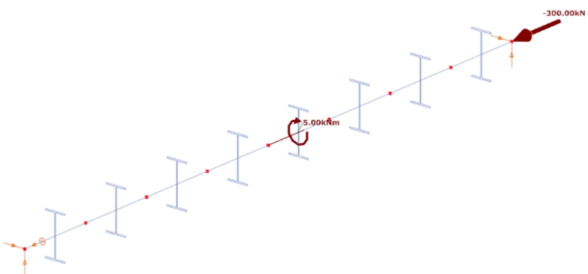


Figure 4 Beam7 finite element model of the simply supported member loaded by torsion moment and compressive force

analysis	results at middle cross-section			
	θ_u (deg)	B (kNm^2)	σ_ω (N/mm^2)	$\sum\sigma_{u,max}$ (N/mm$_2$)
1st order				
- Beam7	5,843	5,040	157,5	197,1
- Shell3	5,854	-	-	196,1
2nd order				
- Beam7	6,884	5,720	178,7	218,3
- Shell3	6,942	-	-	218,0

Table 2 Specific results of the stress analysis

2.3.2 **Example 2**: Global stability analysis

We examine the same structural member defined in Example 1, but the load model now consists of 100 kN concentrated load at the middle of the member and 300 kN compressive axial force (Fig. 5). The global stability analysis supplies the critical load amplifier and the appropriate buckling mode of the member (Fig. 6). Tab. 3 shows the computed critical load amplifiers which were compueted on the general beam-column FE model and on the shell FE modell (Fig.7).

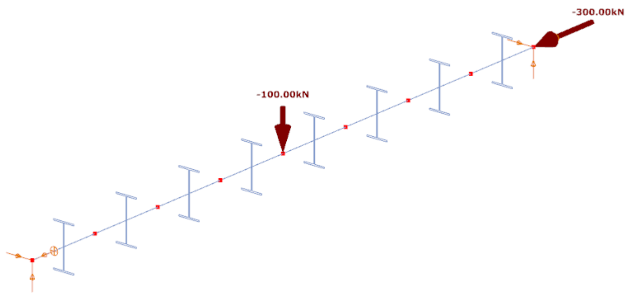


Fig. 5. Beam7 finite element model of the simply supported member loaded by transverse force and uniform compressive force

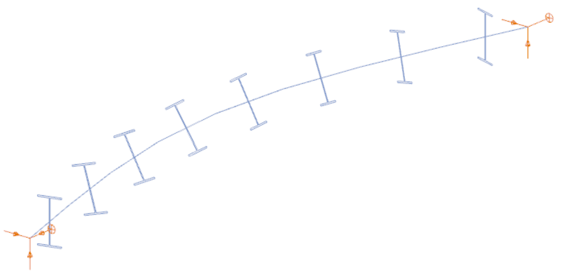


Fig. 6. Global stability analysis by Beam7 model (α_{cr}=1,42)

Fig. 7. Global stability analysis by Shell3 model (α_{cr}=1,40)

FE method	**critical load amplifier (α_{cr})**
Beam7	1,42
Shell3	1,40

Tab. 3. The computed critical load amplifier

3. Analysis of irregular structural members

In Section 2 we have shown that the general beam-column FE method is a very sufficient tool for geometrically nonlinear stress and global stability analysis of uniform structural members. In this Section we will show that this tool is also sufficient in analysis of members with mono-symmetric cross-section (**Example 3**) and/or with tapered web (**Example 4**).

3.1 **Example 3**: Global stability analysis of members with mono-symmetric I section

The efficiency of the general beam-column FE method may be demonstrated by the global stability analysis of a simply supported mono-symmetric I beam published by Mohri et. al. in [5]. They computed the critical moment with theoretically improved and numerical (Abaqus) methods. Fig. 8 shows the *Beam7* model of their benchmark example where the mono-symmetric I section has *150/75-10,3* flanges and *289,3-7,1* web plate (basically it is the simplified cross-sectional model of the IPE300 shape). Fig. 9 shows the lateral torsional buckling mode of the model. Tab. 4 shows Mohri's solutions and the numerical solutions given by the general beam-column FE method.

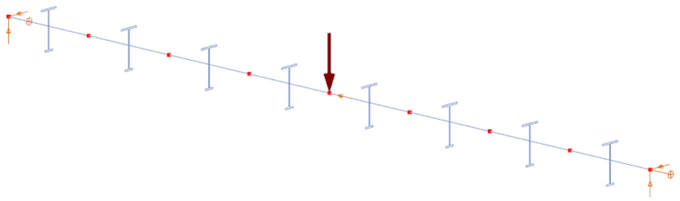


Fig. 8. The Beam7 model of the Mohri's mono-symmetric I beam (L=6.000 mm; load is in the shear centre; E=210.000 N/mm^2)

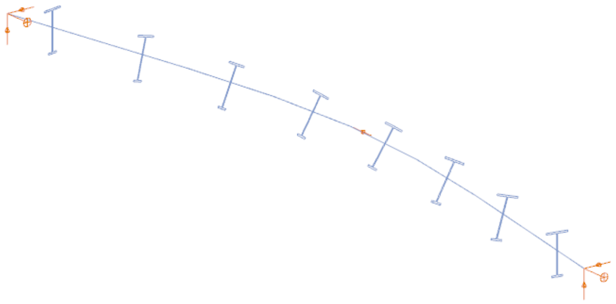


Fig. 9. Lateral torsional buckling mode of the mono-symmetric I beam

method	Critical moment (kNm)	
	down load	**up load**
improved theory *	77,48	54,65
numerical (Abaqus) **	77,41	53,99
numerical (ConSteel) ***	77,98	53,83

* improved theoretical solution by Mohri et.al. [7]
** numerical solution by Mohri et.al. using the S8R5 shell element of the ABAQUS software [7]
*** using the Rajasekaran's general beam-column FE [8]

Tab. 4. Critical moments of the Mohri's benchmark beam computed by different methods

3.3 **Example 4**: Global stability analysis of tapered structural members

The flanges of the tapered member are made of 200-12 plates, the web is made of 588/188-8 plate and the length of the member is 6000 mm. The simply supported member is loaded by 200 kNm concentrated bending moment in the plane of the symmetry and by 100 kN uniform compressive force. The member is modeled with 8 uniform general beam-column elements (**Fig. 10**). The hight of the element is equal to lower hight of the segment. The critical load amplifier and the buckling mode was computed as 1,84 (**Fig. 11**). We computed the critical load amplifier with *Shell3* FE method (**Fig. 12**) also. This method gives 2,03 critical load amplifier (**Tab. 5**).

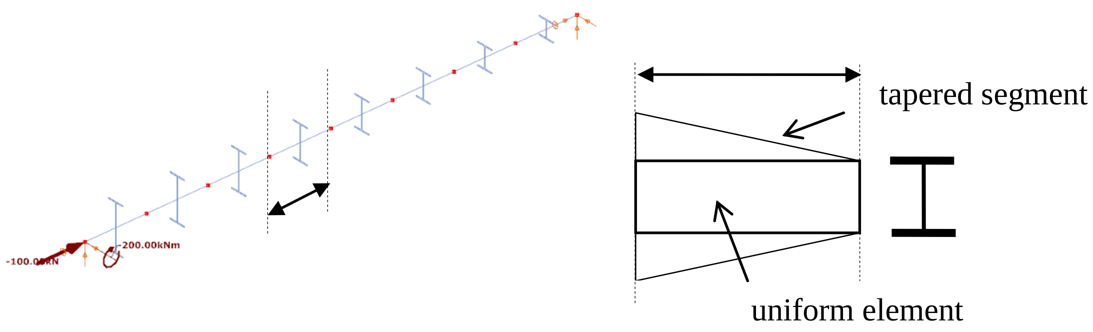


Fig. 10. The Beam7 model of tapered beam (L=6000 mm; normal force is in the centroid; E=210.000 N/mm^2)

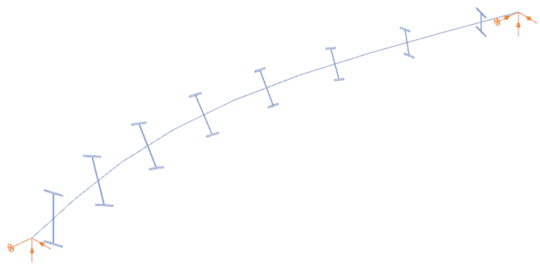


Fig. 11. Lateral torsional buckling mode of the tapered I beam (α_{cr}=1,84)

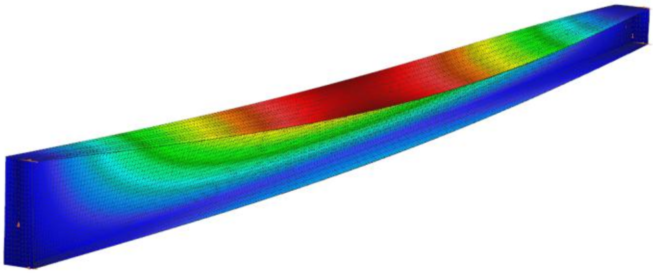


Fig. 12. Buckling mode of the member given by the Shell3 FE element method (α_{cr}=2,03)

method	critical load amplifier
Beam7 (general beam-column FE with 8 elements)	1,84
Shell3 (25 mm)	2,03

Tab. 5. Critical load amplifiers for the tapered models

4. Analysis of complex structures

4.1 Special modeling problems

4.1.1 Transmission of warping

In a more complex structure (ex. frame) the structural members located at a node are on different reference axes (ex. beam-column joint). However, for these nodes the law for the transmission of warping is sophisticated, furthermore, within the beam theory, it is unknown. In order to use the general beam-column FE method for analysis of more complex structures we may apply the simple condition given by Eq. (7). **Example 5** shows that this simple condition for warping gives acceptable analysis if the structural joints are closely rigid.

4.1.2 Load eccentricity

The location of the external forces within the cross-section may drastically influence the critical load amplifier. The general beam-column finite element was derived with the essential assumption that the shear forces are in the shear centre. Consequently, in the initial state of the model the external forces are in the shear centre. To take the load eccentricity into consideration we can connect the loading point and the shear centre with a fictive element (**Fig. 13**). The fictive element is a special and automatically generated finite element which is stiff enough to transmit the effects of the external forces except the warping. **Example 6** illustrates the efficiency of the modeling of load eccentricity using fictive element.

4.1.3 Eccentric elements

The finite element is eccentric if the centroid is out of its reference axis. The eccentricity may be taken into consideration in the geometric transformation matrix of the element. The consequence of the eccentricity is the additional bending moments due to the axial force (**Fig. 14**) and the external torsional moment due to the transverse external forces (**Fig. 15**).

Example 7 illustrates how to use the eccentric element to mode top or bottom steel tapered I members.

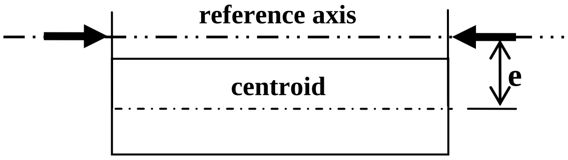


Fig. 14. Axial force effect on the eccentric element

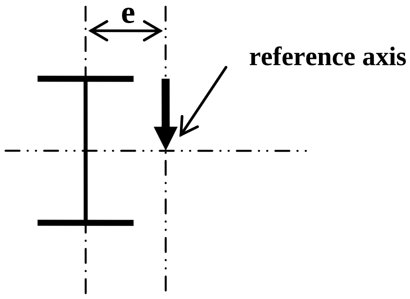


Fig. 15. Transverse force effect on the eccentric element

4.2 Examples

4.2.1 **Example 5**: Global stability analysis of frames

Fig. 16 shows the general beam-column FE model of a simple frame where the beam-to-column joints are continuous in stiffness (moment resistance beam-to-column joints). The frame is supported in the transverse direction at middle and at top of the columns. The frame is load by 100 kN concentrated force at the middle of the beam. Let us compute the critical load amplifier of the frame using the general beam-column FE method and the simplified warping condition given by Eq. (7). As a controlling, we used *Shell3* FE model with different structural solutions for the beam-to-column joints (from semi-rigid joints to rigid joints). **Tab. 6** shows the critical load amplifiers which were computed by different FE methods applying different joint configurations. However, we can take the following conclusions:

- the critical load amplifier depends on the type of the beam-to-column joint configuration
- the result of the general beam-column Fe method is close to the result of the shell FE method if the beam-to-column joint is stiffened (rigid).

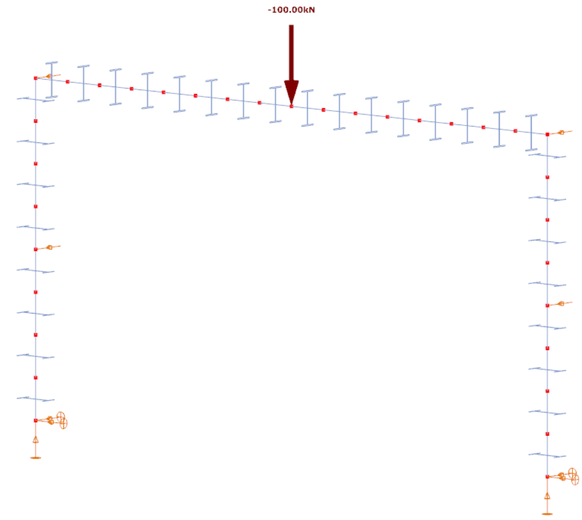


Fig. 16. The Beam7 model of the simple frame structure (span:8000 mm; hieght: 4000 mm;flanges:200-12; web plate:388-8; E=210.000 N/mm^2))

4.2.2 **Example 6**: LTB analysis of a double symmetric beam with an eccentric load

The efficiency of the modeling with general beam-column FE may be illustrated by the global stability analysis of the simply supported symmetric I beam published by Mohri et. al. in [5]. They solved the problem with theoretically improved and numerical methods (Abaqus). The symmetric I section has *150-10,3* flanges and *289,3-7,1* web plate (simplified cross-sectional model of the IPE300 shape). We analyzed this beam by the ConSteel software. We used effective element to model the load eccentricity in case of top and down flange loading. The *Beam7* model is illustrated in **Fig. 17**. We compared the computed critical moments in the **Tab. 7**.

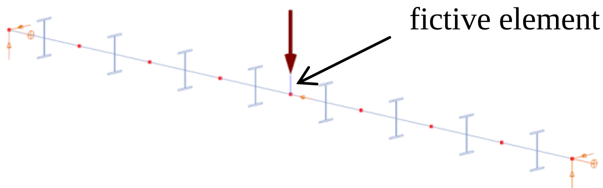


Fig. 17 General beam-column model of the Mohri's beam with top flange down load (E=210.000 N/mm^2; length: 6.000 mm;)

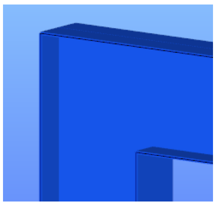
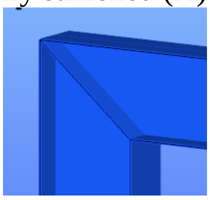
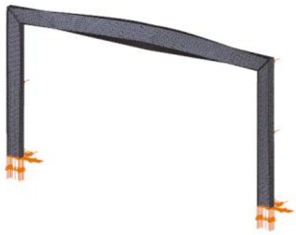
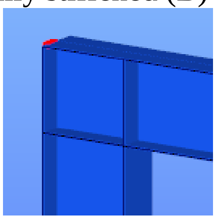
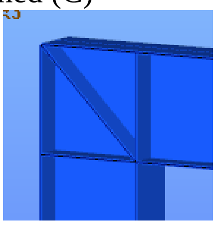
beam-to-column joint model	buckling mode	critical load amplifier (α_{cr})
***Beam7* FE model**		
continuous		**2,66**
continuous but free warping		1,94
***Shell3* FE model**		
unstiffened 		1,38
partially stiffened (A) 		2,12
partially stiffened (B) 		2,10
stiffened (C) 		**2,54**

Table 6 The critical load amplifiers which were computed by different FE methods and by different joint configurations

Solution	Critical moment (kNm)		
	shear centre	top	down
Mohri et.al. (2003) - Abaqus B31OS (Beam7) - Abaqus S8R5 (Shell)	 112,95 -	 79,74 78,45	 159,05 156,57
ConSteel (Beam7)	113,39	80,09	159,53

Tab. 7. Critical moments for the Mohri's beam computed by different methods

4.2.3 **Example 7**: Modeling of tapered frame structure

The span of the symmetric tapered frame is 12.000 mm (between the reference axes of the columns) and the angle of roof is 10 degree. The cross-section at the column base and at the beam-to-beam connection is the same (welded I section with 200-12 flanges and 188/588-8 web plate). The 10 kN/m vertical load is distributed on the reference axes of the beams. **Fig. 18** shows the *Beam7* model of the structure. **Fig. 19** shows the buckling mode of the frame where the critical load amplifier is 7,06. **Fig. 20** shows the buckling mode of the appropriate shell finite element model where the critical load amplifier is 6,36. The shell model contains 12 mm thick base plates and web stiffeners in the beam-to-column and in the beam-to-beam joints. **Tab. 8** shows the specific results of the analysis using different models.

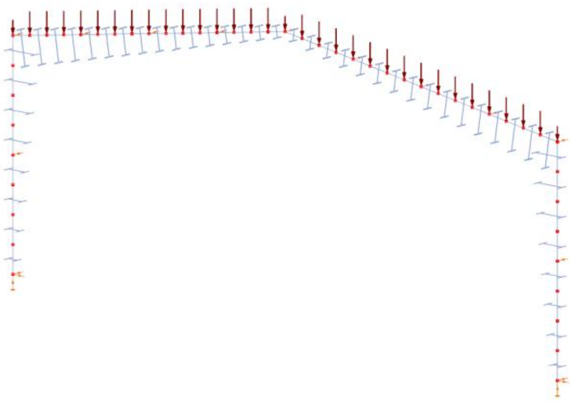


Fig. 18. The Beam7 model of the tapered frame

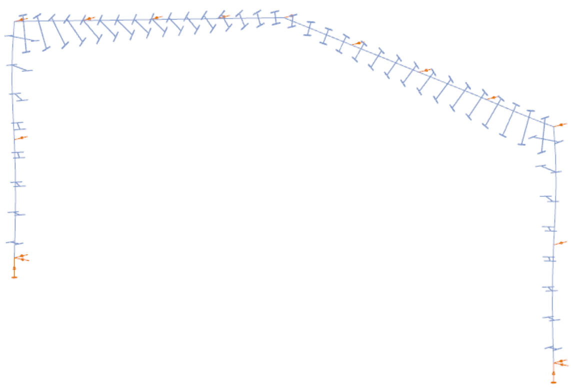


Fig. 19. The buckling mode of the Beam7 model (α_{cr}=7,06)

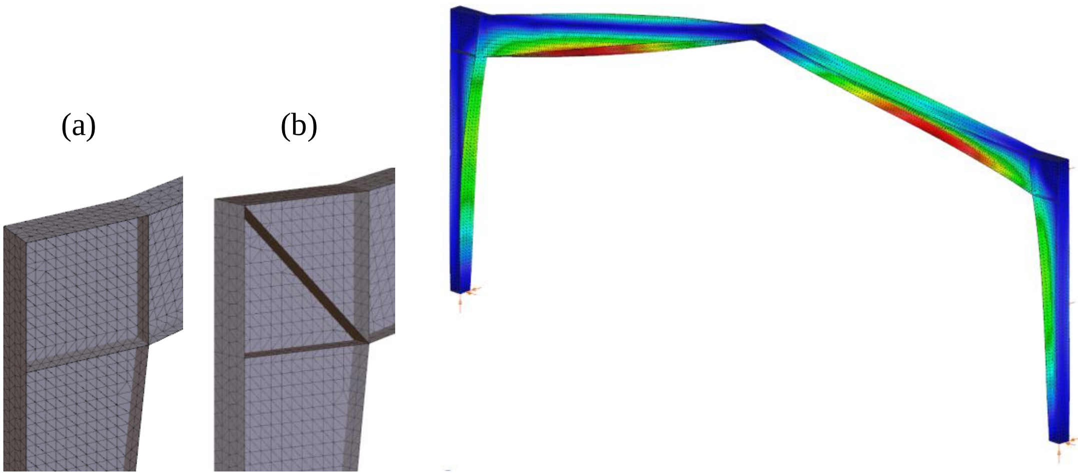


Fig. 20. The buckling mode of the Shell3 model (α_{cr}=6,36) with web stiffeners in the beam-to-column joint

method	**deflection at top of the frame (mm)**	**critical load amplifier**
Beam7	14,48	7,06
Shell3 (50 mm)		
- web stiffeners (a)	12,64	6,52
- web and shear stiffeners (b)	12,17	9,66

Tab. 8. Specific results of the analysis with different models

5. Conclusions

A general beam-column finite element was presented in Section 1. The analysis based on this element provides general elastic second order stress resultants (see Section 2 and Section 3). These stress resultants are required for a comprehensive evaluation of the resistances of any arbitrary cross-section of regular and irregular structural members. The global elastic stability analysis based on this method provides the critical load amplifier which is an essential parameter in the general method for resistances of in-plane structural members and structures (Section 4). The method is allowed when the dominant buckling mode is the lateral torsional buckling or the interaction of flexural buckling and lateral torsional buckling. The method may be sufficiently used for irregular structural members and cross-sections (see Section 4). In this paper we used the ConSteel 4.0 structural design software to illustrate the efficiency of the general design method using general beam-column finite element analysis.

References

[1] EN 1993-1-1 Eurocode 3: Design of steel structures – Part 1.1: General rules and rules for buildings, 2005, pp. 65-67

[2] EN 1993-1-5 Eurocode 3: Design of steel structures – Part 1.5: Plated structural elements, 2006, pp.36-37

[3] Chen, W.F. and Atsuta, T.: Theory of Beam-Columns: Space behavior and design, Vol. 2, McGraw-Hill, 1977, pp. 539-564

[4] www.consteel.hu

[5] Mohri, F., Brouki, A. and Toth, J.C.: Theoretical and numerical stability analyses of unrestrained, mono-symmetric thin-walled beams, Journal of Constructional Steel Research, 59 (2003) 63-90